MEASURES OF DISPERSION

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MEASURES OF DISPERSION

In this topic we study the following measures of dispersion: range and standard deviation. these measures have the same units as that of the observation for example C.M. Hours etc. and the measures are called absolute of measures of dispersion.

of the respective absolute measure.

4.3 RANGE AND COEFFICIENT OF RANGE

Range is a crude measure of dispersion. However, it is the simplest measure and suitable if the extent of variation is small.

Definition : If L is the largest observation and S is the smallest observation then range is the difference between L and S. Thus,

$$\text{Range} = L - S$$

and the corresponding relative measure is

$$\text{Coefficient of range} = \frac{L - S}{L + S}$$

In case of frequency distribution lower limit of first and upper limit of last class intervals are taken to be the smallest and the largest observations respectively.

Note : Requisites of good measures of dispersion are same as those of average.

Merits of Range : (1) It is simple to understand and easy to calculate.

(2) It is rigidly defined.

Demerits of Range : (1) It is not based on all observations. It does not give proper idea regarding variation between the extreme observations.

For example : Range of 0, 3, 5, 200 is same as that of 0, 50, 100, 150, 200, however, variation patterns are different.

(2) It cannot be determined for frequency distribution with open end class.

Applications of Range :

Range is suitable measure of dispersion in case of small group with less variation. (i) It is widely used in the branch of statistics known as Statistical Quality Control. (ii) The changes in prices of shares lowest and highest observations are used. (iii) Temperature at a certain place is recorded using maximum and minimum value. (iv) Range used in medical sciences to check whether blood pressure, hemoglobin count etc. is normal.

Illustration 1 : *Compute range and coefficient of range for the following data :*

100, 24, 14, 105, 21, 35, 106.

Solution : Here,

$$\text{Smallest observation (S)} = 14$$

$$\text{Largest observation (L)} = 106$$

$$\text{Range} = L - S = 106 - 14 = 92$$

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{92}{106+14}$$

$$= \frac{92}{120} = 0.7667$$

Illustration 2 : *Determine the range and the coefficient of range for the following data :*

Electricity consumption per month :	100–150	150–300	300–450	450–600
No. of families :	28	56	43	23

Solution :

$$\text{Range} = \text{Largest observation (L)} - \text{Smallest observation (S)}$$

$$= 600 - 100 = 500$$

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{500}{700} = \frac{5}{7} = 0.7143$$

4.4 STANDARD DEVIATION AND COEFFICIENT OF VARIATION

Here we discuss a measure of dispersion which satisfies most of the requisites of good measure and free from the drawbacks present in the other measures of dispersion.

Definition : The positive square root of mean of squares of the deviations taken from arithmetic mean is called as **standard deviation** (S.D.)

It is denoted by σ (read as sigma, a lower case Greek letter).

Therefore, $\sigma = \sqrt{\dfrac{\sum (x-\bar{x})^2}{n}}$; for individual observations

$= \sqrt{\dfrac{\sum f(x-\bar{x})^2}{N}}$; for frequency distributions

After simplification we can have computational formula for σ in more suitable form as follows :

$\sigma = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$; for individual observations

$= \sqrt{\dfrac{\sum fx^2}{N} - \bar{x}^2}$; for frequency distribution.

where, $\bar{x}$ is a arithmetic mean.

Note : The quantity σ^2 is called as **variance.** Prof. R. A. Fisher has suggested the term variance.

Relative measure of S.D. is called coefficient of variation.

Coefficient of Variation : Prof. Karl Pearson suggested the relative measure of standard deviation. It is called as coefficient of variation (C.V.)

... given by $C.V. = \dfrac{S.D}{|A.M.|} \times 100 = \dfrac{\sigma}{|\bar{x}|} \times 100\%$ ... (4.1)

...fficient of variation is always expressed in percentage.

...marks : (1) R.H.S. of (4.1) includes the multiplier 100, because ...o small in many cases. Thus, for convenience it is multiplied by

Compute S.D. and C.V. for the following data

36, 15, 25, 10, 14

						TOTAL
x	36	15	25	10	14	100
x^2	1296	225	625	100	196	2442

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{100}{5}$$

$$\bar{x} = 20$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

$$\sigma = \sqrt{\frac{2442}{5} - (20)^2}$$

$$\sigma = \sqrt{488.4 - 400}$$

$$\sigma = \sqrt{88.4}$$

$$\boxed{\sigma = 9.402}$$

$$C.V. = \frac{\sigma}{x} \times 100$$

$\boxed{C.V. = 47.01\%}$ $\boxed{C.V. = 47.01\%}$

Compute S.D. and C.V. of marks scored by 10 Candidates given below

54, 61, 64, 69, 58, 56, 49, 57, 55, 50

Solution -: Let

$a = 57$, $d = x - 57$

x	$d = x - a$	d^2
54	$54-57=-3$	9
61	$61-57=4$	16
64	$64-57=7$	49
69	$69-57=12$	144
58	$58-57=1$	1
56	$56-57=-1$	1
49	$49-57=-8$	64
57	$57-57=0$	0
55	$55-57=-2$	4
50	$50-57=-7$	49
Total	$\boxed{d = 3}$	337

$d = 3$

$d^2 = 337$

$n = 10$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{337}{10} - \left(\frac{\Sigma d}{n}\right)^2}$$

$$\sigma = \sqrt{33.7 - \left(\frac{3}{10}\right)^2} \rightarrow (0.3)$$

$$\sigma = \sqrt{33.7 - (0.3)^2}$$

$$\sigma = \sqrt{33.7 - 0.09}$$

$$\sigma = \sqrt{33.61}$$

$$\boxed{\sigma = 5.7974}$$

limit of the items.

4.5 STANDARD DEVIATION OF COMBINED GROUP

Suppose there are two groups with sizes n_1, n_2 having arithmetic means $\bar{x}_1$, $\bar{x}_2$; standard deviations σ_1, σ_2 respectively. Then the mean of combined group is

$$\bar{x}_c = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Let $d_1 = \bar{x}_1 - \bar{x}_c$ and $d_2 = \bar{x}_2 - \bar{x}_c$. Then S.D. of combined group is given by.

$$\sigma_c = \sqrt{\frac{n_1 (\sigma_1^2 + d_1^2) + n_2 (\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

Illustration 6 : *A group of 50 items have mean and standard deviation 61 and 8 respectively. Another group of 100 observations has mean and standard deviation 70 and 9 respectively. Find mean and standard deviation of combined group.*

Solution : We are given that : $n_1 = 50$, $\bar{x}_1 = 61$, $\sigma_1 = 8$, $n_2 = 100$, $\bar{x}_2 = 70$ and $\sigma_2 = 9$. Therefore combined mean

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{(50 \times 61) + (100 \times 70)}{50 + 100} = 67$$

$\therefore$ $d_1 = \bar{x}_1 - \bar{x}_c = 61 - 67 = -6$ and $d_2 = \bar{x}_2 - \bar{x}_c = 70 - 67 = 3.$

$\therefore$ Combined S.D. is

$$\sigma_c = \sqrt{\frac{n_1 \left(\sigma_1^2 + d_1^2\right) + n_2 \left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}}$$

$$\sigma_c = \sqrt{\frac{50\ (64 + 36) + 100 \cdot (81 + 9)}{150}}$$

$$= 9.6609$$

The following data represents the goals scored by two teams in foot ball matches.

Number of goals scored	0	1	2	3	4
No. of Matches by Team A	20	12	8	3	2
No. of Matches by Team B	18	10	7	6	4

Solution :- x = Number of goals scored
f = Number of matches played

Team A

x	f	$f \cdot x$	$fx \times x = fx^2$
0	20	0	$0 \times 0 = 0$
1	12	12	$12 \times 1 = 12$
2	8	16	$16 \times 2 = 32$
3	3	9	$9 \times 3 = 27$
4	2	8	$8 \times 4 = 32$
Total	45	45	103

Team B

x	f	$f \cdot x$	$fx \cdot x = fx^2$
0	18	0	$0 \times 0 = 0$
1	10	10	$10 \times 1 = 10$
2	7	14	$14 \times 2 = 28$
3	6	18	$18 \times 3 = 54$
4	4	16	$16 \times 4 = 64$
Total	45	58	156

Team A

$$\bar{x} = \frac{\Sigma fx}{n}$$

$$\bar{x} = \frac{45}{45}$$

$$\boxed{\bar{x} = 1}$$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{n} - (\bar{x})^2}$$

$$\sigma = \sqrt{\frac{103}{45} - (1)^2}$$

Team B

$$\bar{x} = \frac{\Sigma fx}{n}$$

$$\bar{x} = \frac{58}{45}$$

$$\boxed{\bar{x} = 1.288}$$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{n} - (\bar{x})^2}$$

$$= \sqrt{\frac{156}{45} - (1.28)^2}$$

Team A

$\sigma = \sqrt{2.28 - 1}$

$\sigma = \sqrt{1.28}$

$\sigma = 1.131$

$C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{1.131}{1} \times 100$

$C.V = 1.13 \times 100$

$\boxed{C.V = 113.52\%}$

Team B

$\sigma = \sqrt{3.46 - 1.63}$

$\sigma = \sqrt{1.83}$

$\boxed{\sigma = 1.352}$

$C.V. = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{1.352}{1.288} \times 100$

$C.V. = 1.049 \times 100$

$\boxed{C.V = 104.96\%}$

Standard Deviation Method

Step-1 : Decide assumed mean 'a'
Step-2 : Find the deviations, $d = x - a$
Step-3 : Find the step deviations; $d' = \frac{d}{h}$

Step-4 : Find the sum of d' ~~ad~~ & d'^2

$\Sigma d'$, $\Sigma d'^2$ for individual observations

$\Sigma fd'$, $\Sigma fd'^2$ for frequency distribution

Step-5 : Apply the formula

$$\sigma = \sqrt{\frac{\Sigma d'^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} \times h \text{ for individual obs.}$$

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times h \text{ for frequency distr}$$

$$\underline{\text{Mean}} = a + \frac{\Sigma fd'}{N} \times h$$

$$= \cancel{0}\ 50 + \frac{40}{100} \times 20.$$

$$= 50 + \underline{0.4 \times 20}$$

$$= 50 + 8$$

$$= 58$$

[Standard Deviation] $S.D. = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times h$

$$\sigma = \sqrt{\frac{116}{100} - \left(\frac{40}{100}\right)^2} \times \cancel{h}\ 20$$

$$\sigma = \sqrt{1.16 - 0.16} \times 20$$

$$\sigma = \sqrt{1} \times 20$$

$$\sigma = 1 \times 20$$

$$\boxed{\sigma = 20.}$$

Coefficient of Variation

$$C.V. = \frac{\sigma}{x} \times 100$$

$$C.V. = \frac{20}{58} \times 100$$

$$C.V = 0.3448 \times 100$$

$$\boxed{C.V. = 34.48\%}$$

Calculate the standard Deviation & Coefficient of Variation for the frequency distribution of marks of 100 candidates given below

Marks	0 – 20	20 – 40	40 - 60	60 – 80	80 – 100
frequency	5	12	32	40	11

We use step-deviation method to find σ

Class	Mid Value (x)	frequency (f)	$d' = \frac{x-50}{20}$	$f \times d'$	$d' \times fd' = (f \times d')^2$
0 – 20	10	5	−2	$5 \times -2 = -10$	-10×-2
20 – 40	30	12	−1	$12 \times -1 = -12$	-12×-1
40 – 60	50	32	0	$32 \times 0 = 0$	0×0
60 – 80	70	40	1	$40 \times 1 = 40$	1×40
80 – 100	90	11	2	$11 \times 2 = 22$	2×22
		$N = 100$		$fd' = \cancel{A} = 40$	$\Sigma fd'^2 = 116$
					116

$$d' = \frac{x-a}{20}$$

$$d' = \frac{10-50}{20} \quad d' = \frac{30-50}{20} \quad d' = \frac{70-50}{20} \quad d' = \frac{90-50}{20}$$

$$d' = \frac{-40}{20} \quad d' = \frac{-20}{20} \quad d' = \frac{20}{20} \quad d' = \frac{90-50}{20}$$

$$d' = -2 \quad d' = -1 \quad d' = 1 \quad d' = \frac{40}{20}$$

$$d' = 2$$

Here $a = 50$

$h = 20$

$N = 100$